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# On the Continuity of the Boosts for Each Orbit

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**Abstract.** The possibility of a continuous choice of the boost for each orbit is studied by making use of some mathematical theorems on fibre bundles and it is shown that this choice is possible only for massive particles.

## 1. Introduction

From the Wigner pioneer work, [1] it is well known the characterization of the elementary particles by the unitary irreducible representations of $\tilde{\mathscr{P}}_0$ (universal covering group of $\mathscr{P}_0$). A realization is obtained by the Wigner-Mackey technical procedure of the induced representations (for a modern exposition, see e.g. Simms [2]). For such a realization it is necessary the choice of a point $\hat{p}$ on a certain orbit, and a transformation $L(p)$ of $SL(2, C)$ mapping $\hat{p}$ in a generic point $p$ of this orbit. This mapping must be a Borel mapping, and we can consider this boost as a Borel section of the fibre bundle

$$SL(2, C) \to SL(2, C)/G_{\hat{p}}$$

where $G_p$ denotes the little group of the point $p$.

There are four different kinds of orbits (strata), whose orbits we call $\Omega_m^{\pm}$, $V_0^{\pm}$, $\Omega_{im}$, 0, and whose little groups respectively are $SU(2)$, $\Delta$ (euclidean plane group), $SU(1, 1)$ and $SL(2, C)$ [2].

For the first kind of orbits corresponding to the massive particles the most natural choice of the point $\hat{p}$ is $\hat{p} = (0, 0, 0, m)$ and for $L(p)$, a pure Lorentz transformation because of the polar decomposition of any matrix in $SL(2, C)$. (By using that the unitary matrices lie in the little group of the point $\hat{p}$.) This choice is not only a Borel map but a continuous one, that is to say, a cross section of the bundle.

We show here, by analyzing some topological properties of fibre bundles, that it is not possible to find a cross section in the other cases.

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