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A Formalism for the Investigation of Algebraically Special Metrics. I

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Abstract. A new formalism is proposed for the investigation of algebraically special metrics. Among its advantages are that the essential calculations are co-ordinate free and the equations are gauge invariant. The derived equations are simple in form hence easy to work with and the approach is rich in possibilities not explored by previous techniques.

This paper contains no new results but is an introduction to the technique and a demonstration of its use. New results will be presented in further papers.

Introduction

In the past ten years, several techniques for the investigation of Einstein's equations have appeared, notably those associated with the names of Robinson [1], Bondi *et al.* [2], Newman and Penrose [3], and Debever *et al.* [4]. Most of them share the following properties.

1) They are methods of reducing the Einstein equations to what might be called a minimal set of differential equations (MDE) where, with the exception of asymptotic properties, the analysis ends unless the MDE is soluble.

2) They work very well when applied to algebraically special metrics, but with the exception of the above mentioned asymptotic properties are more or less impotent in the face of an algebraically general metric.

3) They represent a formidable notational barrier to the uninitiate.

In this paper, yet another method is proposed. Since it is designed solely for algebraically special metrics it suffers a priori from defect number two. Similarly defect number three may be said to apply, although adherents of the Newman-Penrose (NP) formalism should find the notation comprehensible on first reading. The advantages claimed for the method are that not only is it an efficient method to arrive at the MDE (possibly the most efficient), but also that its usefulness does not end when these prove insoluble.

The system is an algebra of objects of good spin and boost weights and differential operators which are modified versions of those of