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Self-similar Spacetimes: Geometry and Dynamics*

Douglas M. Eardley
California Institute of Technology, Pasadena, California, USA

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Abstract. The nature and uses of self-similarity in general relativity are discussed. A spacetime may be self-similar (homothetic) along surfaces of any dimensionality, from 1 to 4. A geometric construction is given for all self-similar spacetimes. As an important special case, the "spatially self-similar cosmological models" are introduced, and their dynamical properties are studied in some detail: The initial-value problem is posed, the ADM formulation is established (when applicable), and it is shown that the evolution equations preserve a self-similarity of initial data. The existence of a conserved quantity is deduced from self-similarity. Possible applications to cosmology and singularities are mentioned.

1. Introduction

Similarity solutions in classical hydrodynamics have been a fruitful source of models for physical systems having no intrinsic scale of length, or mass, or time. In (classical) general relativity, the fundamental constants G and c reduce the number of independent physical units to one; take it to be the unit of length. Therefore, the physical notion of self-similarity for spacetime amounts precisely to the geometric notion of *invariance under scale transformations*, as was first pointed out by Cahill and Taub [1].

If a strongly self-gravitating system evolves in size through many orders of magnitude, either expanding or contracting, one might reasonably expect it to "forget" its initial conditions and eventually become scale-invariant. For example, the expansion of the universe from the big bang and the collapse of a star to a singularity might both exhibit self-similarity in some form. This expectation is borne out in most of the popular models for these processes, as will be discussed briefly below. Conversely, one may hope to discover new facts about cosmology and singularities by building new models that presume self-similarity from the out-set.

With these applications in mind, this paper systematically defines and analyzes the notion of self-similarity in the context of general relativity. The immediate goal is a set of tools; physical applications will not be attempted here. The main conclusion is that self-similarity

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