Commun. math. Phys. 37, 257—272 (1974)

On the Equivalence of the Euclidean and Wightman Formulation of Field Theory

V. Glaser
CERN, Geneva, Switzerland

Received December 3, 1973

Abstract. A mistake in the paper [1] on the "Axioms for Euclidean Green's Functions" is corrected in the following sense: thanks to these axioms the Euclidean Schwinger functions S_n can be analytically continued to the corresponding Wightman functions W_n possessing all the correct analyticity properties and satisfying a generalized positivity condition in the complex domain. It is however suggested by the proof that their tempered behaviour near the Minkowski points cannot be guaranteed without additional assumptions [1].

1. Introduction

The very interesting paper on the "Axioms for Euclidean Green's functions" by Osterwalder and Schrader [1] claims to prove the equivalence of the usual Wightman axioms with axioms for the Euclidean Green's functions as formulated in [1]. Unfortunately the crucial Lemma 8.8 of that paper turns out to be wrong [2].

Some years ago, the present author had studied the inter-relation between the positivity condition and the analyticity properties of the Green's functions in momentum space [2]. As pointed out in that paper, the theorems proved there could be easily translated into analogous theorems on the Wightman functions in x space. It turns out that these theorems are essentially sufficient to prove the statements made in [1]; but in a restricted sense. The Euclidean Green's functions satisfying the Osterwalder-Schrader postulates can be shown to be restrictions of functions analytic in the whole Wightman causal domain and to satisfy the positivity condition there in a sense to be presently explained. The author has, however, not been able to show the tempered growth of those analytic functions near the real Minkowski space boundary and he

[1] K. Osterwalder has informed me that he and R. Schrader have arrived independently at the same conclusions. See [7] for an account of their proof. In addition, [7] contains a discussion of conditions which guarantee temperedness of the Wightman functions.

[2] This fact was established by R. Schrader who constructed a counter example to the lemma, inspired by a query from B. Simon who, in his Zurich lectures, had questioned the proof given in [1].