Commun. math. Phys. 37, 93—120 (1974)

The $\lambda\varphi_3^4$ Field Theory in a Finite Volume

Joel Feldman*
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts, USA

Received February 8, 1974

Abstract. The unnormalized doubly cutoff Schwinger functions converge as the ultraviolet cutoff is removed. The limits, the finite volume unnormalized Schwinger functions, are tempered distributions and are C^∞ in the coupling constant. They have asymptotic expansions given by perturbation theory. For λ sufficiently small they can be normalized and then they are the moments of a measure on $\mathscr{S}'_R(\mathbb{R}^3)$.

The $P(\varphi)_2$ models are the best behaved models studied in constructive field theory. The Wightman axioms have been verified for these theories (for weak coupling), firmly establishing their existence, and work related to $P(\varphi)_2$ is now largely aimed at determining physical properties and simplifying earlier proofs. The $\lambda\varphi_3^4$ model, which we are considering in this paper, is the next best behaved boson model. It differs from $P(\varphi)_2$ by having ultraviolet divergences and by requiring ultraviolet divergent mass and wave function as well as vacuum energy renormalizations. Work on $\lambda\varphi_3^4$ is still aimed at establishing its existence. The principal progress in this direction has been the proof of the existence [2] and semiboundedness [3] of the spatially cutoff Hamiltonian. In this paper we use the methods of [3] to show that the (unnormalized) spatially cutoff Schwinger functions exist, are tempered distributions, and are C^∞ in the coupling constant. If λ is small we can normalize the Schwinger functions and then they are the moments of a probability measure on $\mathscr{S}'_R(\mathbb{R}^3)$. The next step in the program might involve the use of methods developed for $P(\varphi)_2$ (see [4, 5]) to take the infinite volume limit and verify the Wightman axioms. Another open problem is that of determining if, as conjectured, the free and (spatially cutoff) interacting measures are mutually singular.

Readers are referred to [3] and [5] for further background material, notation and references and for details related to the inductive expansion.

We will be concerned solely with the Euclidean approach to φ_3^4. The free theory is given on the path space $L^2(\mathscr{S}'_R(\mathbb{R}^3), dq_0)$ where dq_0 is the Gaussian measure with mean zero and covariance $\mu^{-2} = (-\Delta + 1)^{-1}$.

* Supported in part by the National Science Foundation under Grant GP 40354X.