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Analytic Renormalization Using Many Space-Time Dimensions

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Abstract. A renormalization method originally proposed by Ashmore is reformulated and shown to be, in fact, a renormalization. The method involves use of different complex dimensions associated with various subgraphs of a graph, and appears to combine the best features of complex dimensional and analytic renormalization.

In an earlier paper Ashmore [1] proposed an ingenious synthesis of analytic and complex-dimensional renormalization, based on the use of distinct complex dimensions associated with various subgraphs of a Feynman graph. Unfortunately, several errors in this paper necessitate a fairly complete reformulation of the method; one such reformulation is presented here. Our regularization procedure (§ 1) is the same as that of [1], although we give it in an x-space version which is needed later. The renormalization procedure, also defined in § 1, differs from that of [1] by taking a different set of parameters as basic. In § 2 we prove that the renormalization has an additive structure, as desired.

I. Regularization and Renormalization

Let G be a fixed 2-connected Feynman graph, with vertex set $U_0 = \{V_1, \ldots, V_m\}$ and line set $\mathscr{L}$. We will ignore spin in this paper (although the methods given extend readily) and thus assume that a line $\ell \in \mathscr{L}$ has propagator

$$\tilde{\Delta}_\ell(p) = -i(m_\ell^2 - p^2 - i\,0)^{-1},$$

with $m_\ell > 0$. We first establish our terminology.

If H is a subgraph of G, $N(H)$ denotes the number of loops of H, $\mathscr{L}(H)$ the set of lines of H. H naturally decomposes into 2-connected subgraphs and single lines joining them; these are called the *pieces* of H. A *generalized vertex* U is a subset of U_0, and $G(U)$ denotes the graph which contains all lines joining any pair of vertices in U; we will drop the distinction between U and $G(U)$ when no confusion can arise. In

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