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Haag-Ruelle Approximation of Collision States

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Abstract. We investigate the rate of convergence of the Haag-Ruelle approximation $\Psi(t)$ at large times t for arbitrary collision states Ψ with finite energy. An improved estimate of the norm distance $\|\Psi - \Psi(t)\|$ is given. In particular for states Ψ with smooth asymptotic wave functions it turns out that $\|\Psi - \Psi(t)\|$ approaches 0 almost like $t^{-3/4}$.

I. Introduction

The fundamental work of Haag [1] and Ruelle [2] established the existence of states which can be interpreted as asymptotic particle configurations within the framework of quantum field theory. Since this ingenious construction is by now well known it may suffice to sketch the procedure briefly: given any incoming or outgoing particle configuration one can construct sequences of vectors $\Psi(t)$ in a Hilbert space $\mathscr{H}$ by applying products of suitably chosen almost local one-particle creation operators at time t to the vacuum vector. These Haag-Ruelle approximations converge strongly in the limit of large negative and positive times and the limit vectors Ψ^{in} and Ψ^{out} correspond to the given incoming and outgoing particle configuration respectively.

One might think that one can forget about the approximations $\Psi(t)$ once one has constructed the collision states Ψ^{in}, Ψ^{out} since all the information relevant for physics is contained in matrix elements which are computable from these vectors. However, for some problems it is sufficient and much simpler to consider the approximations of a given collision state instead of the state itself. Several interesting results in collision theory have been derived from the well known kinematical properties of the vectors $\Psi(t)$ at finite times t and the convergence behaviour of the sequences $\Psi(t)$. For example, Araki and Haag showed that local observables provide a direct interpretation of scattering states as asymptotic particle configurations [3]. For a detailed summary of results of more technical nature see the lecture notes of Araki [4] and Hepp [5].

In all these investigations, the crucial point is to derive an adequate estimate for the norm distances $\|\Psi^{\text{in}} - \Psi(t)\|$ and $\|\Psi^{\text{out}} - \Psi(t)\|$ at large