

# Block-Spins Interactions in the Ising Model

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**Abstract.** We investigate the interactions between block-spins in the large magnetic field region for the nearest neighbour,  $d$ -dimensional, Ising model.

## 1. Introduction

We try to analyse a number of properties of an Ising model in the attempt to make some rigorous statements about the theories connected with the renormalization group approach to critical phenomena [1].

We cannot make any statements about the critical point; however we can say something away from it.

One could, with reasons, argue that this region is not interesting since there is “nothing” to predict. Nevertheless it seems to us of interest to try to put “known things” in a form which could suggest some precise mathematical conjectures about the critical region.

We show that, if the system is away from the critical point, some consequences of renormalization group arguments can be reduced to very general theorems of probability.

Although we point out, in the final comments, some features which might be of interest in the critical region we do not go into an analysis of the relation between our work and the renormalization group approach [1].

In Section 2 we describe some notations and the main result. Since, at first sight, our theorem looks rather obscure, we clarify it with a series of remarks.

In Section 3 we describe the main tools used to prove our theorem. The technique is basically due to Ruelle [2] who worked it out for the continuum case; it was extended to the lattice case in [3] (but the combinatorics was wrong; the right combinatorial algebra can be found, for instance, in [4]).

The main theorem is proven in Section 4 using a technique already employed in [5] for proving some central limit theorems and extended in [4]: we use the notations and results proven in [4].