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Exact Models of Charged Black Holes

I. Geometry of Totally Geodesic Null Hypersurface*

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Abstract. Inner geometry and embedding formulas for a totally geodesic null hypersurface $\mathscr{M}$ in an electrovacuum space-time are given. The structure of all possible symmetry groups of the geometry is described in case that the space-like sections $\mathscr{S}$ of $\mathscr{M}$ are compact orientable surfaces and $\mathscr{M}$ is, topologically, $\mathscr{S} \times R^1$. The result is $\mathscr{G} = \tilde{\mathscr{G}} \times \mathscr{H}$, where $\tilde{\mathscr{G}}$ are the well-known isometry groups of $\mathscr{S}$, and $\mathscr{H}$ is an at most two-dimensional group acting along rays, which is fully specified in the paper. It is not shown that all these symmetry types exist, but this will be done in the next papers where all horizons of a given symmetry type will be explicitly written down.

1. Introduction

The interest in the theory of collapsed stars – black holes – has recently increased, not only because of a theoretical appeal of these objects. It seems that some phenomena observed in the sky, e.g. the large X-ray source in Cygnus [1], can most naturally be explained by the accretion of matter onto a black hole [2].

In the present and subsequent papers, we describe a way of obtaining exact models of charged (electrically and magnetically) black holes surrounded by matter and charge currents without solving explicitly the Einstein-Maxwell equations. The method is akin to that of [3], [4], and [5], in which the initial value constraints of Einstein's equations are solved along a space-like hypersurface of time symmetry, or, geometrically, along a totally geodesic space-like hypersurface. What we are doing is to solve the initial value constraint of the *characteristic* Cauchy problem for Einstein-Maxwell equations along a totally geodesic *null* hypersurface (TGNH). The relation to black holes is as follows. Hawking [6] has shown that the area of black holes cannot decrease. In a limiting case, it will remain constant, the convergence ϱ and the shear σ will be zero and the horizon will be TGNH; such a black hole cannot swallow anything and can be, therefore, called "a fasting black hole" (a name proposed by Kundt). In the "thermodynamic" language of [7], one could also call the holes "adiabatic", but we should like to

* Devoted to Professor André Mercier who is sixty in April, 1973.