Commun. math. Phys. 33, 53—60 (1973)

The Classical Limit of n-Vector Spin Models

Colin J. Thompson
Mathematics Department, University of Melbourne, Victoria, Australia

Howard Silver
Physics Department, University of Melbourne, Victoria, Australia

Received February 20, 1973

Abstract. It is proved that the free energy of a system of n-dimensional spins with Kac type potential is equal, in the infinite range zero strength limit, to the free energy of the corresponding Curie-Weiss system in which every spin interacts equally with every other spin.

1. Introduction

In 1966 Lebowitz and Penrose [1] proved that the free energy of a classical system of particles in ν-dimensions with pair potential $v(\boldsymbol{r})$ of Kac type,

$$v(\boldsymbol{r}) = q(\boldsymbol{r}) + \gamma^{\nu} \varrho(\gamma \boldsymbol{r}) \tag{1.1}$$

approaches the van der Waals free energy with Maxwell construction in the limit $\gamma \to 0+$ (after the thermodynamic limit) provided the short range repulsive (hard core) part of the potential $q(\boldsymbol{r})$ and the long range attractive part of the potential $\gamma^{\nu}\varrho(\gamma \boldsymbol{r})$ satisfied certain conditions (stated in [1]).

It is not difficult, as suggested by Lebowitz and Penrose, to extend the analysis to Ising ferromagnets (or equivalently, attractive lattice gases) and show that the classical Curie-Weiss theory of magnetism can be obtained from a $\gamma \to 0+$ limit [2].

Here we consider the n-vector model, first introduced by Stanley [3], composed of a set of N, n-dimensional spins

$$\boldsymbol{S}_i = (S_{i1}, S_{i2}, \ldots, S_{in}), \quad i = 1, 2, \ldots, N \tag{1.2}$$

occupying the vertices of a ν-dimensional lattice, with norm

$$\|\boldsymbol{S}_i\| = \left(\sum_{k=1}^{n} S_{ik}^2 \right)^{1/2} = n^{1/2} \tag{1.3}$$

and with interaction energy

$$E = - \sum_{1 \leqq i < j \leqq N} \varrho_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - \boldsymbol{H} \cdot \sum_{i=1}^{N} \boldsymbol{S}_i , \tag{1.4}$$