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Investigation of the Critical Point in Models of the Type of Dyson's Hierarchical Models

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Abstract. We consider the classical spin models where the Hamiltonians are small modifications of the Hamiltonians of Dyson's hierarchical models. Under some assumptions we investigate rigorously the neighbourhood of the critical point and find the critical indices. It follows that in the cases under consideration phenomenological Landau's theory of phase transitions is valid.

Introduction

In classical lattice ferromagnets the critical temperature T_{cr} separates the domains with zero and non-zero spontaneous magnetization. The behaviour of different thermodynamical parameters near T_{cr} was considered rigorously for the two-dimensional Ising model using Onsager's exact solution (see [1, 2]), and some other models (see [3]) also using the exact formula for the free energy.

Recently Dyson introduced so-called hierarchical models immitating in many respects the lattice systems with pairwise long-range power interaction. Under several natural assumptions Dyson proved that the spontaneous magnetization in his models is non-zero for sufficiently large β ([4–6]).

We consider in this paper a class of models slightly generalizing hierarchical models and find rigorously under some conditions critical indices for them. Recently some non-rigorous results in this direction were obtained by Baker [7]. Our case corresponds to the "gaussian case" of his paper. Closely related results were presented also in [8] (see also [9]).

Now we want to describe briefly our main results. We impose certain conditions on the distribution of the mean spin in a finite volume of fixed size for an interval of temperatures. Under these conditions we prove the existence of a critical temperature T_{cr} inside this interval and for $T = T_{cr}$ we establish the limit distribution for the mean spin which is a gaussian distribution with a non-usual normalization. This permits us to find the asymptotic of binary correlation functions for $T = T_{cr}$.

For $T > T_{cr}$ we obtain the asymptotic expression for the susceptibility and binary correlation functions. For $T < T_{cr}$ we find an asymptotic