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Spacetimes with Killing Tensors*

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Abstract. For Einstein-Maxwell fields for which the Weyl spinor is of type $\{2, 2\}$, and the electromagnetic field spinor is of type $\{1, 1\}$ with its principal null directions coaligned with those of the Weyl spinor, the integrability conditions for the existence of a certain valence two Killing tensor are shown to reduce to a simple criterion involving the ratio of the amplitude of the Weyl spinor to the amplitude of a certain test solution of the spin two zero restmass field equations. The charged Kerr solution provides an example of a spacetime for which the criterion is satisfied; the charged C-metric provides an example for which it is not.

I. Introduction

The geodesic equation is said to admit a *quadratic first integral* if there exists on spacetime a symmetric tensor field K_{ab} which satisfies the *Killing equation*[1]

$$\nabla_{(a}K_{bc)} = 0 .$$

Let t^a be tangent to an affinely parametrized geodesic,

$$t^a \nabla_a t^b = 0 ;$$

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[1] The Battelle conventions of Penrose [1] are used here for the denotation and manipulation of tensor and spinor indices.