Commun. math. Phys. 32, 107—118 (1973)

Two Remarks on Extremal Equilibrium States

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Received February 20, 1973

Abstract. First it is shown that each extremal equilibrium state is representable as limit of Gibbs states in finite volumes, and that an analogous statement holds for extremal invariant equilibrium states. Secondly we prove that for negative pair interactions only one equilibrium state exists which minimizes (resp. maximizes) the particle density, but that in general there are more than two extremal invariant equilibrium states with the same particle density. In this context, periodic interactions are studied.

1. Notations[1]

Let us consider the ν-dimensional cubic lattice $T = \mathbb{Z}^\nu$ ($\nu \in \mathbb{N}$) and the set $\mathfrak{B}$ of all non-void subsets V of T with finite cardinality $|V|$. Denote by $\bar{V}$ the complement of V. Given any $V \subset T$, we consider the set $C_V = \{0, 1\}^V$ of all configurations of particles in V; in particular, we set $C = C_T$. Further on we shall use the projections $\pi_V^W : C_W \to C_V$ ($V \subset W \subset T$) and $\pi_V = \pi_V^T$.

We assume that the particles are subjected to a pairwise interaction which is described by a function U in

$$\mathfrak{U} = \{ U \in \mathbb{R}^T : U(0) = 0,\, U(t) = U(-t)\, (t \in T),\, \sum_{t \in T} |U(t)| < \infty \}.$$

Given in addition a "chemical potential" $\mu \in \mathbb{R}$, for all $V \in \mathfrak{B}$ and $\bar{c} \in C_{\bar{V}}$ the *Gibbs Distribution* $q_{V/\bar{c}} = q_{V/\bar{c}}^{\mu, U}$ *on* C_V *under the condition* $\bar{c}$ is defined by

$$\begin{aligned} U_{V/\bar{c}}(c) = -\mu \sum_{t \in V} c_t + \tfrac{1}{2} \sum_{s,t \in V} c_s c_t U(s-t) \\ + \sum_{s \in V,\, t \in \bar{V}} c_s \bar{c}_t U(s-t) \end{aligned} \tag{1.1}$$

$$Z_{V/\bar{c}} = \sum_{c \in C_V} \exp[-U_{V/\bar{c}}(c)], \quad q_{V/\bar{c}}(c) = Z_{V/\bar{c}}^{-1} \exp[-U_{V/\bar{c}}(c)],$$

where c is any configuration in V.

[1] For a proof of the interspersed facts, see [3], e.g.