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Essential Self-Adjointness of Operators in Ordered Hilbert Space

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Abstract. Let $H_0 \geqq 0$ be a self-adjoint operator acting in a space $L^2(M, \mu)$. It is assumed that $H_0 e = 0$, where e is strictly positive, and that $\exp(-tH_0)$ is positivity preserving for $t \geqq 0$. Let V be a real function on M such that its positive part is in $L^2(M, e^2 \mu)$ and its negative part is relatively small with respect to H_0. Then $H = H_0 + V$ is essentially self-adjoint on the intersection of the domains of H_0 and V. This result is applied to Schrödinger operators and to quantum field Hamiltonians.

I. Introduction

Let $H_0 \geqq 0$ and V be self-adjoint operators. If V is sufficiently regular and if the negative part of V is suitably small, then the (quadratic form) sum $H = H_0 + V$ is a uniquely defined self-adjoint operator [6; Chapter VI]. There need be no restriction on the size of the positive part of V. However it does not follow that there are very many vectors in the intersection of the domains of H_0 and V. Additional conditions are needed to ensure that H be essentially self-adjoint on the intersection of the domains, and that is the subject of this paper.

The main results are the essential self-adjointness theorem for operators acting in an ordered Hilbert space (Theorem 4.4) and its application to Schrödinger operators (Theorem 5.2). This application gives a particularly simple proof of essential self-adjointness for Schrödinger operators without use of partial differential equation methods. The proof of the theorem is based on a theory of contractive semigroups and an extension of a lemma of Davies [1].

A theory of essential self-adjointness using L^p space methods and hypercontractive semigroups was developed for use in quantum field theory [10, 12, 13, 15, 5] and was applied to Schrödinger operators by Simon [13]. Simon treats only potentials V which are bounded below. They are required to be locally in L^2 and to satisfy a growth condition at infinity. By using partial differential equation techniques Davies [1] was able to deal with V which are unbounded below. The positive part of V is required to be locally in L^p for some $p > \frac{n}{2}$, $p \geqq 2$ (where n is the