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The BBGKY Hierarchy in Quantum Statistical Mechanics*

Sezar Fesciyan

Belfer Graduate School of Science, Yeshiva University, New York, USA

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Abstract. It is shown that the infinite volume limit of the equilibrium reduced density matrices, shown by Ginibre to exist at low densities, satisfy the quantum time independent BBGKY hierarchy of equations. This extends analogous results for classical systems due to Gallavotti.

Introduction

Gallavotti [1] has shown rigorously that the infinite volume equilibrium correlation functions for a classical system satisfy the time independent BBGKY hierarchy of equations for a large class of potentials whenever the activity is small enough. For this purpose he made use of results due to Groeneveld, Penrose and Ruelle (see Chapter 4 in [2]) relating to the existence of the correlation functions in the thermodynamic limit. We made use of the corresponding results due to Ginibre [3, 4] relating to the existence of the quantum counterparts of the correlation functions, namely the reduced density matrices (RDM), and a method suggested by Gallavotti to prove that the time independent form of the quantum version of the BBGKY are satisfied by the infinite volume RDM. We treat the two cases of classical Boltzmann Statistics (C.S.) and Quantum Statistics (Q.S.), both Bose and Fermi, separately.

We are in general interested in a grand ensemble of systems of identical particles, not necessarily in equilibrium, enclosed in a finite box Λ, and interaction via two-body forces so that the Hamiltonian of the system with N particles in Λ has the form

$$H_N = -\sum_{1 \leqq i \leqq N} V_{x_i}^2 + \sum_{1 \leqq i < j \leqq N} \Phi(x_i - x_j) \tag{1.1}$$

where $2m = \hbar = 1$, $x_i \in \Lambda \subset R^\nu$, $V_{x_i} \equiv \dfrac{\partial}{\partial x_i}$ and ν is the dimensionality of space.

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