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General Relativistic Shock Waves in Fluids for which Pressure Equals Energy Density

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Abstract. A shock wave in a self-gravitating fluid obeying the equation of state: pressure equal to energy density is shown to travel with the velocity of light in a space-time determined by the Einstein field equations. The jump conditions that must be satisfied by the hydrodynamic variables are derived and discussed as are those that must be satisfied by the metric tensor and its derivatives. The latter conditions are obtained by using a variational principle.

1. Introduction

The preceding paper [1] discussed some solutions of the Einstein field equations for the case where the source of the gravitational field was a perfect fluid described by the stress energy tensor

$$T^{\mu\nu} = (w + p)\, u^\mu u^\nu - p q^{\mu\nu} \tag{1.1}$$

with u^μ the normalized four-velocity vector, that is, with

$$u^\mu u_\mu = 1\,, \tag{1.2}$$

w the rest energy density and p the pressure. It was also assumed that the fluid obeyed the "extreme" equation of state

$$w = p\,. \tag{1.3}$$

It is the purpose of this paper to discuss the jump conditions that must be satisfied across a singular hypersurface, such as a shock wave, in the space-time determined by the Einstein field equations when the source of the gravitational field is the extreme fluid described by Eqs. (1.1) through (1.3). These jump conditions are of two sorts: (1) conditions that must be satisfied by the hydrodynamic variables, to so-called Rankine-Hugoniot equations and (2) conditions that must be satisfied by the metric tensor and its derivatives.