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Central Decomposition of Invariant States Applications to the Groups of Time Translations and of Euclidean Transformations in Algebraic Field Theory

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Abstract. With $\mathfrak{A}$ a C^*-algebra with unit and $g \in G \to \alpha_g$ a homomorphic map of a group G into the automorphism group of G, the central measure μ_Φ of a state Φ of $\mathfrak{A}$ is invariant under the action of G (in the state space of $\mathfrak{A}$) iff Φ is α-invariant. Furthermore if the pair $\{\mathfrak{A}, G\}$ is asymptotically abelian, Φ is ergodic iff μ_Φ is ergodic. Transitive ergodic states (corresponding to transitive central measures) are centrally decomposed into primary states whose isotropy groups form a conjugacy class of subgroups. If G is locally compact and acts continuously on $\mathfrak{A}$, the associated covariant representations of $\{\mathfrak{A}, \alpha\}$ are those induced by such subgroups. Transitive states under time-translations must be primary if required to be stable. The last section offers a complete classification of the isotropy groups of the primary states occurring in the central decomposition of euclidean transitive ergodic invariant states.

Introduction

The general setting of this paper is the one encountered in algebraic field theory: we are given a C^*-algebra $\mathfrak{A}$ (in physics the "quasi-local algebra", norm completion of the set of local obervables) and a locally compact group G acting as automorphisms of $\mathfrak{A}$ (one of the invariance groups of the physical theory). We recall that the states of $\mathfrak{A}$ (normalized positive functionals) are interpreted as the states of the physical system. The states of $\mathfrak{A}$ invariant under G are of particular interest both mathematically and physically. Mathematically they provide a "non commutative generalization" of the invariant measures basic in ergodic