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The Phase Separation Line in the Two-Dimensional Ising Model*

GIOVANNI GALLAVOTTI
I.H.E.S. Bures sur Yvette, France

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Abstract. We prove that, at low temperature, the line of separation between the two pure phases shows large fluctuations in shape. This implies the translation invariance of the correlation functions associated with some non translation invariant boundary conditions and should be a peculiarity of the dimensionality of the model.

1. The Line of Separation

It has recently been conjectured that the surface of separation between two pure phases is, at low temperature and for short range potential models, rigid in the case of a 3-dimensional model and non rigid in 2-dimensional models [1, 2].

In this paper we prove the truth of the conjecture in the 2-dimensional Ising model.

The precise meaning of what "surface of separation" and "rigid" mean will be given below and has already been discussed in the literature [3].

Let Ω be a $N \times N$ square lattice centered at the origin: let $i = 1, 2, \ldots, N^2$ be a label for the center of each unit square composing Ω. We assume that on each site $i \in \Omega$ is located a spin $\sigma_i = \pm 1$ and that the energy of a spin configuration $\varrho = (\sigma_1, \ldots, \sigma_{N^2})$ is given by:

$$H_N(\sigma) = -\frac{1}{2} \sum_{\langle ij \rangle} \sigma_i \sigma_j - \frac{1}{2} \sum_{i \in \partial^+ \Omega} \sigma_i + \frac{1}{2} \sum_{i \in \partial^- \Omega} \sigma_i \tag{1.1}$$

where $\sum_{\langle ij \rangle}$ means, as usual, sum over the pairs of nearest neighbour couples of points in Ω and $\partial^+ \Omega$ ($\partial^- \Omega$) denote the points adjacent to the upper half (lower half) of the boundary $\partial \Omega$ of Ω.

The physical meaning of (1.1) is that $H_N(\sigma)$ corresponds to the energy of a configuration of spins interacting through a nearest neighbour pair potential and, also, interacting with a set of external fixed spins adjacent

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