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Dynamics of a Rigid Test Body in Curved Space-Time*

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Abstract. A covariant canonical formulation of the motion of a rigid test body in a curved space-time is obtained from a suitable Cartan form θ on the tangent bundle $T\mathscr{P}$ of the bundle of Lorentz frames $\mathscr{P}$ over the space-time manifold V. The form θ (essentially equivalent to a Lagrangean) is chosen in close analogy to the corresponding 1-form in the classical Newtonian model of a rigid body and is very simple in terms of the natural geometrical structure of $\mathscr{P}$. The presymplectic manifold $(T\mathscr{P}, d\theta)$ then serves as evolution manifold of the system. One obtains the equations of motion and also a uniquely defined Poisson bracket on the set of observables defined as real valued functions on the manifold of motions. The rigid body interacts with the space-time curvature only via its spin in the same way as a spinning particle. Quadrupole and higher multipole interactions with the space-time curvature are not considered in this model.

1. Introduction

The equations of motion of spinning particles, gyroscopes and rigid test bodies in curved space-time have been extensively discussed from different points of view (see mainly Suttorp and de Groot [25] and Dixon [5] for a historical review of the earlier literature). They have received added interest recently in view of the proposed gyroscope experiments ([21, 8, 17, 18]).

These equations have first been obtained by various generalizations of classical Newtonian equations which were, however, always somewhat arbitrary and lead to many controversies. Then Papapetrou [20] derived them from the conservation law for the stress-energy tensor of an extended body, an approach that was improved by Tulcyjew [27], Taub [26] and others until Dixon [5–7] and later more elegantly Madore [16] obtained a new and consistent set of equations, after the question of existence and uniqueness of a center of mass worldline was settled (Beiglböck [3]).

In this paper it is shown how Dixon's equations (specialized to a rigid test body freely falling in an exterior gravitational field, with quadrupole

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