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Hydrogen Atom in the Friedman Universe

EKKEHARD NOWOTNY
Lehrstuhl für theoretische Physik der Universität Konstanz

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Abstract. Within the framework of general relativity the Dirac equation for the hydrogen atom is given in case of a spatially isotropic and homogeneous expanding space-time (Robertson Walker metric). In the special case of the static, closed 3-dimensional spherical space (Einstein Universe) we get a continuous energy spectrum for the H-atom.

Introduction

In Einstein's gravitational theory the existence of so-called standard measures (standard rods, standard clocks) is assumed, which allow to measure explicitly the structure of the 4-dimensional space-time. In general, one supposes that the rods and clocks of the microphysics represents such standard measures.

On the other hand, one should emphasize that also the microphysic is embedded in the space-time and hence is influenced by its structure. Nevertheless, the above assumption seems to be justified in so far as in general a noticeable disturbance of the microphysical standard measures by the metric field is to be expected only if the gradients of the metric inside the system are comparable with the interaction forces. But this will be the case only under extreme conditions.

However, these considerations should be taken carefully. Peres [1], [2], and Callaway [3] have analyzed the influence of the space-curvature on the hydrogen spectrum caused by the mass and charge of the hydrogen nucleus. They got a continuous spectrum for the H-atom with normalized wave functions; until today, the resolution of this paradox has not been found.

Furthermore, an influence of the topology of space-time on the microphysics and therefore also on the microphysical standard measure should be expected. Already 1936 Taub [4], 1938 Schrödinger [5] and 1946 Infeld and Schild [6] have shown that in the case of free electrons embedded in a closed 3-dimensional space of positive curvature there exist differences in the solutions of the Dirac equation for the spherical