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Stability of Homogeneous Universes*

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Abstract. The stability of a class of homogeneous cosmological models is investigated. It is shown that the perturbation problem for six such universes can be reduced to a system of ordinary differential equations. The time development of the perturbations is such that they remain finite at all times for which the unperturbed metric is non-singular.

I. Introduction

In a previous paper [1] the stability of the Taub universe was analyzed and shown to reduce to a system of ordinary differential equations. In this paper we generalize that result to universes whose 3-surfaces of homogeneity admit a simply transitive, 3-parameter group of motions of Bianchi types I, II, VII_0, VIII, and IX, as well as to the Kantowski-Sachs universe [2]. Except for the Kantowski-Sachs case, which is discussed in Appendix B, all these universes belong to "class A" in the classification of homogeneous cosmological models given by Ellis and McCallum [3]. These are non-rotating universes with the flow vector of matter (assumed perfect fluid) orthogonal to the surfaces of homogeneity. The group structure is of the form

$$[X_1, X_2] = N_3 X_3\,, \quad [X_2, X_3] = N_1 X_1\,, \quad [X_3, X_1] = N_2 X_2 \tag{1.1}$$

where the N_α[1] can be chosen to equal 0 or ± 1.

If we define the 1-forms ω^α by $\langle \omega^\alpha, X_\beta \rangle = \delta^\alpha_\beta$, we can write the metric for these universes as

$$ds^2 = dt^2 - A(\omega^1)^2 - C(\omega^2)^2 - B(\omega^3)^2 \tag{1.2}$$

where A, C, and B are functions of the time, t, to be determined by Einstein's field equations (see [3], Section 4, for justification of this form of the metric). We will further restrict our universes by requiring *local rotational symmetry* [3, 4]. This is equivalent to demanding $A = C$ and $N_1 = N_2$ and is made in order that the "Laplacian operator" $g^{\alpha\beta} X_\alpha X_\beta$ separates in the coordinates used to express the X_α (see Appendix A). We thus consider groups satisfying

$$[X_1, X_2] = N X_3\,, \quad [X_2, X_3] = n X_1\,, \quad [X_3, X_1] = n X_2 \tag{1.3}$$

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1 Greek indices have the range 1, 2, 3; Latin 0, 1, 2, 3.