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Spin Waves and the BCS Model

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Abstract. We discuss the behaviour of the BCS model in the limit of infinitely many degrees of freedom. A new limiting procedure, based on spin waves, is proposed, by which the usual convergence difficulties can be overcome.

Introduction

This article is concerned with the behaviour of the Bardeen-Cooper-Schrieffer model [1] in the limit of infinitely many degrees of freedom. Since this problem has already been extensively studied by several authors [2–5], some explanation is needed for the publication of a new paper on this subject.

The method used by the above authors is, in essence, the following: for any finite number, say Ω, of degrees of freedom, the system is determined by a C^*-algebra $\mathfrak{A}_\Omega$ and a Hamiltonian H_Ω. The algebras $\mathfrak{A}_\Omega$ form an ascending series,

$$\mathfrak{A}_\Omega \subseteq \mathfrak{A}_{\Omega'}$$

if $\Omega < \Omega'$, thus it is possible to define a new C^*-algebra $\mathfrak{A}_\infty$ by

$$\mathfrak{A}_\infty = \text{norm completion of } \bigcup_\Omega \mathfrak{A}_\Omega$$

$\mathfrak{A}_\infty$ is the smallest C^*-algebra containing all $\mathfrak{A}_\Omega$.

Now one constructs suitable representations π of $\mathfrak{A}_\infty$ – mostly the thermodynamic representations [6] which are readily obtained using the results of Thirring and Bogoliubov, Jr. [7] – and asks the following questions:

i) does $\pi(H_\Omega)$ converge, at least on a dense set?
ii) does $\pi(\exp iH_\Omega t)$ converge towards a unitary operator?
iii) does, for $S \in \pi(\mathfrak{A}_\infty)$

$$\pi(\exp iH_\Omega t)\, S\pi(\exp - iH_\Omega t)$$

converge and determine an automorphism of the algebra $\mathfrak{A}_\infty$? (This automorphism may, of course, be representation-dependent.)