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Small-Distance-Behaviour Analysis and Wilson Expansions

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Abstract. A previously described method to obtain the asymptotic forms of vertex functions at large momenta is, with the help of Wilson operator product expansion formulas, extended to momenta where the vertex functions of the zero-mass theory underlying the asymptotic forms are infrared singular. To obtain from asymptotic forms information on asymptotic behaviour requires assumptions on the behaviour of the zero-mass theory in the limit of infinite dilatation. One particular set of assumptions is discussed and found to pass a simple consistency test; this set of assumptions leads to power laws, or slight modifications thereof, with coupling-constant-independent exponents. The detailed discussion is given for the ϕ^4 model.

Introduction

Of great interest in quantum field theory is the asymptotic behaviour of vertex functions (VFs) (i.e., the amputated one-particle irreducible parts of Green's functions)[1] $\Gamma(p_1 \ldots p_{2n})$, $\Sigma p = 0$, as the momenta become large. We formalize this question by investigating

$$f(\lambda) = \Gamma((\lambda p_1 + r_1) \ldots (\lambda p_{2n} + r_{2n})), \Sigma p = \Sigma r = 0, \tag{0.1}$$

for large λ. In perturbation theory one obtains [1–3], formally,

$$f(\lambda) = \lambda^{2\alpha} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} f_{kl} \lambda^{-k} (\ln \lambda)^l \tag{0.2}$$

where the even integer 2α is Weinberg's [1] asymptotic exponent and depends on n and the momenta $p_1 \ldots p_{2n}$, and each f_{kl} is an infinite power series in g. Keeping in (0.2) only the $k=0$ part defines the asymptotic form (AF) of $f(\lambda)$; keeping also the parts with $k \leqq m$ defines the AF of m^{th} degree[2].

[1] For conciseness, we write formulas for the $g\phi^4$ theory, the considerations of this introduction and in essence of the whole paper being valid, however, for all renormalizable field theories.

[2] If we use the term AF without specifying the degree, we mean the AF of zeroth degree.