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The Relativistic Problem of Motion in Co-moving Co-ordinates*

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Abstract. The role of co-moving atlases is discussed in connection with a possible formulation of the problem of motion in General Relativity. The concept of co-moving scheme is defined and applied to various cases of physical interest. In particular in the Einstein-Maxwell case, we derive a general uniqueness proof for the Maxwell equations.

The dynamical meaning of the equation $T^{ij}{}_{/\!/j}=0$ is proved, and a scheme for the solution of the problem of motion in co-moving co-ordinates is proposed.

§ 1. Introduction

In a previous paper [1] we have shown how the problem of motion for a material continuum in General Relativity may be conveniently formulated in terms of the projection operators associated to the congruence Γ of stream-lines of the continuum itself.

Our results were obtained under the hypothesis $\Gamma \in (C_1, \tilde{C}_2)$ [1], a condition which is general enough to include every physical situation, with the only exception of shock waves [2–4].

However, almost all problems of actual physical interest may be treated under the stronger condition $\Gamma \in (C_2, \tilde{C}_4)$.

In this case, it is always possible to choose local co-ordinates (x^1, x^2, x^3, x^4) in the space-time manifold $\mathscr{V}_4$ in such a way that the curves of Γ have local equation $x^4 = \text{var}$.

We call such a collection of local co-ordinates a co-moving atlas.

The use of co-moving atlases simplifies the structure of the problem of motion, and makes the whole formalism introduced in [1] more transparent.

The object of the present paper is to discuss this particular view point, and to indicate its implications.

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[1] The notation $\tilde{C}_n$ stands for "piecewise C_n"; the notation $(C_m, \tilde{C}_n)\,(m<n)$ for "C_m, piecewise C_n".