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An Ising Ferromagnet with Discontinuous Long-Range Order

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Abstract. An infinite one-dimensional Ising ferromagnet M with long-range interactions is constructed and proved to have the following properties. (1) M has an order-disorder phase transition at a finite temperature. (2) Any Ising ferromagnet of the same structure as M, but with interactions tending to zero with distance more rapidly than those of M, cannot have a phase-transition. (3) The long-range-order parameter (thermal average of the spin-spin correlation at infinite distance) jumps discontinuously from zero in the disordered phase to a finite value in the ordered phase. All three properties have been conjectured by Anderson and Thouless to hold for a particular Ising ferromagnet which is relevant to the theory of the Kondo effect. Although M is not identical to Anderson's model, the results proved for M support the validity of the physical arguments of Anderson and Thouless.

I. Introduction

Anderson and his colleagues [1–4] have discussed the Ising ferromagnet with Hamiltonian

$$H = -J \sum_{n<m}\sum (m-n)^{-2} \mu_m \mu_n , \qquad J > 0 , \tag{1.1}$$

an infinite one-dimensional linear chain of spins whose state is specified by the dichotomic variables $\mu_n = \pm 1$. Using asymptotic estimates which are probably correct although not rigorous, Anderson concludes that the system (1.1), which we shall call the "Anderson Model," has an order-disorder phase transition at a temperature T_c given approximately by

$$\beta_c J = 0 \cdot 63 , \qquad \beta_c = (k T_c)^{-1} . \tag{1.2}$$

It is known (Dyson [5]; Ruelle [6]) that the model (1.1) with α replacing 2 in the exponent has a phase-transition for $1 < \alpha < 2$ but not for $\alpha > 2$ or $\alpha \leqq 1$. The Anderson model is thus a delicate border-line case, and a rigorous proof that it has a phase-transition is much to be desired. In this paper we do not supply such a proof. The best we can do in this direction is to prove

Theorem 1. *The linear Ising ferromagnet with Hamiltonian*

$$H = -J \sum_{n<m}\sum (m-n)^{-2} \log\log(m-n+3)\, \mu_m \mu_n , \qquad J > 0 , \tag{1.3}$$

has a phase-transition.