Commun. math. Phys. 20, 205—219 (1971)

Probabilistic Metric Spaces and Hysteresis Systems

T. ERBER

Department of Physics, Illinois Institute of Technology, Chicago

B. SCHWEIZER

Department of Mathematics and Statistics, University of Massachusetts, Amherst

A. SKLAR

Department of Mathematics, Illinois Institute of Technology, Chicago

Received July 6, 1970

Abstract. A phenomenological theory of simple hysteresis is constructed with the aid of certain concepts from the theory of probabilistic metric spaces. The predicted forms of the dependence of average energy loss per hysteresis cycle on the maximum excursion of the hysteresis coordinate agree well with experimental results.

1. Introduction

During the last half-century numerous attempts have been made to devise appropriate measures for sets of curves in ordinary or phase spaces. Much of this effort has had its roots in the work of N. Wiener on Brownian motion [1, 2] and, in common with Wiener's work, is motivated by problems derived from physics. Indeed, two of the better-known methods are due principally to physicists: these are the methods of lattice enumerations [3], and Feynman path-integrals [4]. In the first, the formidable convergence difficulties that inevitably arise were averted by replacing continuous paths by finite sequences of lattice points; in the second, convergence problems were essentially brushed aside, and results obtained formally under the guidance of physical intuition.

Many of these convergence and general measure-theoretic difficulties arise from the necessity of taking into account paths whose behavior may be indecorous in various ways: nonrectifiable paths; paths that backtrack and retrace themselves, perhaps infinitely often; self-intersecting paths; paths without tangents; etc. Such paths do in fact correspond to current idealizations of the behavior of physical systems in certain situations, e.g., Brownian motion, Zitterbewegung, and turbulence. However, there are non-trivial physical systems in which such mathematical intricacies are ruled out. Specifically, in systems undergoing