Commun. math. Phys. 19, 327–331 (1970)
© by Springer-Verlag 1970

Classical Schwinger Terms

David G. Boulware*
Physics Department, University of Washington, Seattle, Washington

S. Deser**
Physics Department, Brandeis University, Waltham, Massachusetts

Received July 3, 1970

Abstract. Schwinger terms must be present in equal time ground state Poisson brackets among currents unless the latter are stationary in that state.

A general result in quantum mechanics is the necessary non-vanishing of the equal time commutator of any operator with its time derivative. Only the existence of a unique lowest energy state and the positivity of the Hilbert space metric enter into the derivation which uses the equation of motion

$$\dot{A} \equiv \frac{\partial}{\partial t} A(t) = -i[A(t), H], \tag{1}$$

to evaluate the vacuum expectation value[1]

$$\begin{aligned} \langle 0 | [A(t), \dot{A}] | 0 \rangle &= -i \langle 0 | [A(t), [A(t), H]] | 0 \rangle \\ &= 2i \langle 0 | A(t) \, H \, A(t) | 0 \rangle \\ &= 2i \sum_n E_n | \langle n | A(0) | 0 \rangle |^2 . \end{aligned} \tag{2}$$

The final summation is positive semi-definite since E_n is; it can vanish only if the vacuum is an eigenstate of A. An immediate consequence in local field theory is that the equal time commutator $[j^0(r), j^k(r')]$ of a conserved current $j^\mu(x)$ cannot vanish [1, 2], since its divergence

$$\langle 0 | [j^0(\boldsymbol{r}), \ \nabla'_k j^k(\boldsymbol{r}')] | 0 \rangle = - \langle 0 | [j^0(\boldsymbol{r}), \ \partial_0 j^0(\boldsymbol{r}')] | 0 \rangle \tag{3}$$

must be non zero (and a non vanishing local current cannot annihilate the vacuum) [3]. A more precise result is that, assuming the equal time

* Supported in part by U.S.A.E.C. Contract AT(45-1)1388 B.
** Supported in part by U.S.A.F. OAR under OSR Grant 70-1864.

1 The zero of energy is chosen so that the vacuum lowest energy state has zero energy. The vacuum state is denoted by $|0\rangle$.