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Coherent Photon States and Spectral Condition

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Abstract. A criterion is derived for the existence of a selfadjoint and semibounded momentum operator in a non-Fock representation of the free photon field given by a coherent state. The representation of the translation group is constructed and it is shown that the rotation group, hence the homogeneous Lorentz group, cannot be unitarily implemented in the so-called infrared sectors.

1. Introduction

It has been stated many times that the theory of soft photon emission primarily deals with non-Fock representations of the outgoing free electromagnetic field. As is well known, the use of strange, so-called infrared representations is necessitated by the singular behavior of the convection current of the accelerated charged particle at the vertex of the light cone in momentum space [1]. It has further been realized that coherent states [2, 3], suitably generalized to account for an "infinite photon number", not only provide a useful tool to cope with infrared problems in quantum electrodynamics [4–7], but also arise naturally in a study of classical external currents [8, 9], the Bloch-Nordsieck [10] and the Pauli-Fierz model [11, 12]. A deeper reason for the occurrence of coherent states may be seen in their intimate connection with the Poisson process which governs the photon emission.

Any coherent state, considered as a functional on the quantized radiation field, gives rise to a representation of that field and, clearly, the existence of a momentum operator in that representation which is inevitable for a spectral analysis of the photon emission restricts the class of admissible states. Therefore, we aim to find a necessary and sufficient condition for a coherent state to define a sector that is covariant with respect to the space-time translations [1]. Assuming this condition to be satisfied, it will become evident that in general the energy is not bounded from below and, as a consequence, cannot be uniquely defined. A second criterion then assures that a momentum operator with physical spectrum

[1] We mention that our discussion will have various points of contact with the theory of direct product representations [13], exponential Hilbert spaces [14], and infinitely divisible group representations [15], but will not draw on any results of these works.