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Spatial Representation of Groups of Automorphisms of von Neumann Algebras with Properly Infinite Commutant

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Abstract

Theorem. Let a topological group G be represented $(a \to \phi_a)$ by $*$-automorphisms of a von Neumann algebra $\boldsymbol{R}$ acting on a separable Hilbert space $\boldsymbol{H}$. Suppose that

(a) G is locally compact and separable,

(b) $\boldsymbol{R}'$ is properly infinite,

(c) for any $T \in \boldsymbol{R}$, $x, y \in \boldsymbol{H}$ the function

$$a \to \langle \phi_a(T)x, y \rangle_{\boldsymbol{H}}$$

is measurable on G. Then there exists a strongly continuous unitary representation of G on $\boldsymbol{H}$, $a \to U_a$, such that for $T \in \boldsymbol{R}$, $a \in G$,

$$\phi_a(T) = U_a T U_a^* .$$

Let $\boldsymbol{R}$ be a von Neumann algebra acting on a separable Hilbert space $\boldsymbol{H}$. Let G be a topological group and the map $a \to \phi_a (a \in G)$ a representation of G by $*$-automorphisms of $\boldsymbol{R}$.

General Problem. When is there a strongly continuous unitary representation, $a \to U_a$, of G on $\boldsymbol{H}$ such that for $a \in G$, $T \in \boldsymbol{R}$, $\phi_a(T) = U_a T U_a^*$? The theorem below gives an affirmative answer for a large class of von Neumann algebras. This result may be of use in Quantum Mechanics. This theorem is a generalization of a theorem of Kallman [2]. The author wishes to express his gratitude to Kallman for suggesting this problem.

Theorem. *In the context of the general problem stated above, suppose that*

(a) *G is locally compact and separable,*

(b) *the commutant of $\boldsymbol{R}$ is a properly infinite von Neumann algebra,*

(c) *(weak measurability) for any $T \in \boldsymbol{R}$, $x, y \in \boldsymbol{H}$, the function*

$$a \to \langle \phi_a(T)x, y \rangle_{\boldsymbol{H}}$$

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