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A Class of Homogeneous Cosmological Models

II. Observations

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Abstract. This paper discusses the application of geometric optics to the study of observational properties of cosmological models examined in a previous paper. A number of results concerning these properties are derived, the most interesting of which is the invariance of observational relations under certain discrete isotropy groups. Closed form expressions are obtained in certain cases.

1. Introduction

This paper discusses the observational properties of a class of homogeneous cosmological models studied in previous papers [1–3]. These are spacetimes which satisfy Einstein's field equations for a perfect fluid and which admit a three-parameter group of motions simply-transitive on spacelike sections (surfaces of homogeneity) orthogonal to the fluid flow vector[1], u^i. They are therefore universes homogeneous in the rest-space of any fundamental observer.

In this paper we will quote freely from the results of the earlier work. The matter in these spaces has no rotation or acceleration. One can choose coordinates $\{t, x_\nu\}$ such that $\{x_\nu\}$ are comoving coordinates, $\{t = \text{constant}\}$ are the surfaces of homogeneity, and t is the proper time along the world-lines of the matter (Latin indices run from 0 to 3, Greek from 1 to 3; $a, b, c \ldots \alpha, \beta \ldots$ will be used for components referred to an orthonormal tetrad $\{\boldsymbol{e}_a\}$ with $\boldsymbol{e}_0 = \boldsymbol{u}$; $i, j, k \ldots$ will be used for coordinate components). $\{\boldsymbol{e}_\kappa\}$ span the tangent plane to the surface of homogeneity at each point. The signature is $+2$ and u^a is normalised ($u^a u_a = -1$). The first derivatives of u_a are determined by the expansion tensor θ_{ab},

$$u_{a;b} = \theta_{ab}; \quad \theta_{ab} = \theta_{(ab)}; \quad \theta_{ab}u^b = 0. \tag{1.1}$$

[1] Spacetimes admitting a multiply-transitive group acting on such three-dimensional spacelike surfaces belong to the class of L.R.S. (locally rotationally symmetric) spaces [4,49]. The only such spacetimes not admitting a simply-transitive subgroup acting on these surfaces are those of Case I of Kantowski and Sachs [5].