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Exponential Representations of the Canonical Commutation Relations

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Abstract. A class of representations of the canonical commutation relations is investigated. These representations, which are called exponential representations, are given by explicit formulas. Exponential representations are thus comparable to tensor product representations in that one may compute useful criteria concerning various properties. In particular, they are all locally Fock, and non-trivial exponential representations are globally disjoint from the Fock representation. Also, a sufficient condition is obtained for two exponential representations not to be disjoint. An example is furnished by Glimm's model for the $:\Phi^4:$ interaction for boson fields in three space-time dimensions.

I. Introduction

In this paper we investigate a certain class of representations of the canonical commutation relations. Our representations will be called *exponential* Weyl systems. A representation of the canonical commutation relations, or a *Weyl system*, is a map $f \to W(f)$ from a complex inner product space J to unitary operators on a complex Hilbert space H, such that

$$W(f)\, W(g) = e^{i\,\mathrm{Im}(f,g)/2}\, W(f+g)$$

(the Weyl relations), and $t \to (\phi, W(tf)\psi)$ is continuous at $t=0$. If $\{f_j\}$ is an orthonormal basis of J, then

$$W(sf_j) = e^{isQ_j}, \qquad W(tf_k) = e^{itP_k},$$

by Stone's theorem, where Q_j, P_k are self-adjoint. The Weyl relations are

$$e^{isQ_j}\, e^{itP_k} = e^{ist\delta_{jk}}\, e^{itP_k}\, e^{isQ_j},$$

which is an exponentiated version of

$$Q_j P_k - P_k Q_j = i\delta_{jk}.$$