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On the Mathematical Description of Quantized Fields

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Abstract. We start from Haag's proposal to describe quantum fields at a point, corresponding to the heuristic description by means of their matrix elements $(A(x)\,\Phi|\,\Psi)$ between vectors of a dense linear manifold D of the Hilbert space. We particularize this idea, so that the sesquilinear functional that describes the field at a point may be considered as an element of the sequential completion of a space of operators, endowed with a suitable "D-weak" topology.

It is shown that any Wightman field may be described in this way, as a rather elementary consequence of the existence of a translation invariant cyclic vacuum. Furthermore the field turns out to be an infinitely differentiable function of space and time.

§ 1. Introduction

It has been first proposed by Haag (1963, Ref. [1]) that the quantum field $A(x)$ at any point x should be described by means of a sesquilinear functional over a dense linear manifold D of the Hilbert space H, corresponding to the heuristically defined mapping $(\Phi, \Psi) \to (A(x)\,\Phi|\,\Psi)$.

The work of Kristensen, Mejlbo, and Poulsen (1965, Ref. [2]) and Grossmann (1967, Ref. [3]) may be considered as providing two different topological particularizations of this idea. These authors show, among other things, that free fields and Wick products of them may be described, in their sense, as infinitely differentiable functions of space and time.

In this work we consider another topological particularization of Haag's proposal, which is very simple, and we show that essentially any Wightman field may be described in this way as an infinitely differentiable function of space and time. We only require the existence of a cyclic translation-invariant vacuum.

§ 2. C_D: A non Commutative Topological $*$-Algebra of Unbounded Operators

Definition 1. *We use the following notation: R means the real field, M the (Minkowski) space-time, which in this paper may be identified with the m-dimensional real space R^m (where the physically most interesting value of m is 4), C means the complex field.*

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