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Anisotropic Stresses in Homogeneous Cosmologies

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Abstract. Recently a certain class of homogeneous world models filled with perfect fluid have been discussed, [1, 2]. The corresponding results when anisotropic stresses are included are now examined.

This paper discusses general-relativistic spacetimes admitting a 3-parameter group of motions acting simply-transitively on spacelike surfaces of homogeneity such that the tangent vectors of the normal congruence are Ricci eigenvectors. In previous papers [1, 2] the normals u^a were further assumed to be the fluid flow vectors for a perfect fluid, and the aim of this paper is to investigate the validity of previous results when this restriction is relaxed. Physically this will allow us to consider a fluid with 4-velocity u^a and anisotropic stresses, but with no net energy flux relative to the fluid flow.

The calculation techniques and group classification follow [1]. The results of [1], Section 2, remain valid, and in particular, $\omega = \dot{u} = 0$ and ∂_α applied to any covariantly defined quantity gives zero. The energy-momentum tensor has the form [9],

$$T^{ab} = \mu u^a u^b + p h^{ab} + \pi^{ab} \tag{1}$$

where $\pi^{ab} = \pi^{(ab)}$, $\pi^a{}_a = \pi^{ab} u_b = 0$. T^{ab} must have the form (1) because u^a is assumed to be a Ricci eigenvector. If we interpret u^a as the mean fluid flow vector then μ is the energy density, p the pressure, and π_{ab} the anisotropic stress measured in the rest frame of u^a. The contracted Bianchi identities $T^{ab}{}_{;b} = 0$ read,

$$\dot{\mu} + \mu\theta + (p h^{ab} + \pi^{ab}) \left(\frac{1}{3} \theta h_{ab} + \sigma_{ab} \right) = 0 \,, \tag{2}$$

$$3a^\nu \pi_{\nu\alpha} + \pi^{\gamma\mu} \varepsilon_{\gamma\alpha\tau} n_\mu{}^\tau = 0 \,. \tag{3}$$