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# The van der Waals Limit for Classical Systems

## II. Existence and Continuity of the Canonical Pressure

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**Abstract.** For a $\nu$-dimensional system of particles with the two-body potential $q(\boldsymbol{r}) + \gamma^\nu K(\gamma \boldsymbol{r})$ and density $\varrho$, it is proved under fairly weak conditions on $q$ and $K$ that the canonical pressure $\pi(\varrho, \gamma)$ and chemical potential $\mu(\varrho, \gamma)$ tend to definite limits when $\gamma \to 0$. The limiting functions are absolutely continuous and are given in terms of the derivative of the limiting free energy density $a(\varrho, 0+) \equiv \lim_{\gamma \to 0} a(\varrho, \gamma)$ which was found in Part I.

## I. Introduction

In Part I of these papers [1] we considered the free energy density $a(\varrho, \gamma)$ of a $\nu$-dimensional system of particles with the two-body potential

$$q(\boldsymbol{r}) + \gamma^\nu K(\gamma \boldsymbol{r}) \tag{1.1}$$

and density $\varrho$. (We assume there is no external field in the present paper). Under fairly weak conditions on $q$ and $K$ we proved that the *van der Waals limit* $a(\varrho, 0+) \equiv \lim_{\gamma \to 0} a(\varrho, \gamma)$ exists and is given by a variational formula.

In the present paper we consider the canonical chemical potential

$$\mu(\varrho, \gamma) \equiv \frac{\partial}{\partial \varrho} a(\varrho, \gamma) \tag{1.2}$$

and the canonical pressure

$$\pi(\varrho, \gamma) \equiv \left( \varrho \frac{\partial}{\partial \varrho} - 1 \right) a(\varrho, \gamma) \tag{1.3}$$

for the same system. The existence of these functions was proved by Dobrushin and Minlos [2] (see also [3]). We prove that *their van der Waals limits*

$$\mu(\varrho, 0+) \equiv \lim_{\gamma \to 0} \mu(\varrho, \gamma), \tag{1.4}$$

$$\pi(\varrho, 0+) \equiv \lim_{\gamma \to 0} \pi(\varrho, \gamma) \tag{1.5}$$