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A $\lambda\phi^{2n}$ Field Theory without Cutoffs*

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Abstract. We consider a self-interacting scalar boson field in two-dimensional space-time with self-interaction given by an arbitrary Wick polynomial of even degree in the field. It is shown that the field theory can be constructed in a Hilbert space of physical states. The hamiltonian is a positive self-adjoint operator possessing a physical vacuum. The method of proof consists of imposing and then removing three cutoffs: a box cutoff, an ultraviolet cutoff, and a space cutoff. As the first two are removed the resolvents of the cutoff hamiltonians converge uniformly and this leads to the self-adjointness of the spatially cutoff hamiltonian.

§ 1. Introduction and Discussion of Results

In this paper we consider the self-interacting boson field in two-dimensional space-time with hamiltonian given formally as

$$H_{\text{formal}} = H_0 + \lambda \int :P(\phi(x)): dx \tag{1.1}$$

where H_0 is the free hamiltonian for the mass $m > 0$,

P is an arbitrary polynomial of even degree,
$P(y) = y^{2n} + b_{2n-1} y^{2n-1} + \cdots + b_0$,

and

λ, the coupling constant, is taken equal to 1 in this paper unless otherwise indicated.

From perturbation theory considerations we expect this model, when rigorously treated, to provide a Lorentz-covariant local quantum field theory with nontrivial scattering. This is the motivation for this study in which we take the first steps towards a field theory.

As has been emphasized by Wightman (see, for example, [21]), a formal expression for the hamiltonian like (1.1) is highly singular and must be "cutoff" or "butchered" if we wish to use the interaction picture or to work in Fock space. For instance, a version of Haag's theorem [21, § VI] states that either we must destroy the translation invariance of the density $:P(\phi(x)):$ in (1.1) or else we must work with a strange representation of the commutation relations (and cannot use Fock space).

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