Infinitely Divisible Representations and Positive Definite Functions on a Compact Group

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Abstract. A complete description of the infinitely divisible positive definite functions on a compact group is given. Their relation to infinitely divisible representations is also discussed.

§ 1. Introduction

In certain problems of quantum field theory it is necessary to define the notion of a continuous tensor product of Hilbert spaces. In his attempt to construct such tensor products Streater [3] has introduced the notion of an infinitely divisible group representation. These in turn are described by infinitely divisible positive definite functions on the group. If the group is abelian such positive definite functions are in one to one correspondence with the so called infinitely divisible distributions on the character group. The reader may find a detailed account of these in [1]. Here we shall give a complete description of the infinitely divisible positive definite functions on a compact group and describe some of the infinitely divisible representations.

§ 2. Infinitely Divisible Group Representations and Positive Definite Functions

Throughout this paper we shall denote by G a fixed compact metric group. By a representation of G we mean a continuous unitary representation of G in a complex separable Hilbert space with an inner product $(.,.)$. Let $T^{(k)}, k = 1, 2, \ldots$ be an enumeration of all the inequivalent irreducible representations (up to equivalence). Suppose $T^{(k)}$ operates in a Hilbert space V_k of dimension d_k. Let $\varepsilon_1^k, \varepsilon_2^k, \ldots, \varepsilon_{d_k}^k$ be a