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An Algebraic Spectrum Condition

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Abstract. A condition, necessary and sufficient for the existence of a vacuum representation with positive energy of the quasilocal algebra, is formulated.

Most postulates of axiomatic quantum field theory can be translated easily into the language of C^*-algebras [1]. A remarkable exception is the usual spectrum condition. The required algebraic formulation has to assure the existence of a vacuum representation of the quasilocal algebra, for which the energy-momentum spectrum is contained in the future cone $\bar{V}_+$. Such a representation will be called a "positive vacuum representation" [4]. Algebraic spectrum conditions have been formulated by Doplicher [2], Montvay [3], and Borchers [4]. In this note, we will propose another condition of this type.

Consider a C^*-algebra $\mathfrak{A}$, called quasilocal algebra[1], and a representation $x \to \alpha_x$ of four-dimensional translations x by automorphisms α_x of $\mathfrak{A}$. This representation shall be strongly continuous, i.e.,

$$\lim_{x \to 0} \|\alpha_x A - A\| = 0$$

for any $A \in \mathfrak{A}$. Then $\mathfrak{A}$ contains, as a norm-dense invariant sub-$*$-algebra $\mathfrak{A}^{(1)}$, the set of all $A \in \mathfrak{A}$ for which

$$\operatorname*{norm-lim}_{\tau \to 0} \frac{1}{\tau} (\alpha_{\tau a} A - A) \underset{\text{df.}}{=} D_a A$$

exists for all four-vectors a [5].

A positive linear functional φ on $\mathfrak{A}$, normalized to $\|\varphi\| = 1$, is called a state. Then, for arbitrary state φ and $A \in \mathfrak{A}^{(1)}$, the functions

$$\hat{\varphi}(\tau \mid A, a) \underset{\text{df.}}{=} \varphi(A^* \alpha_{\tau a} A)$$

are differentiable with respect to τ. Denote by E_+ the set of all states φ for which

$$\frac{1}{i} \frac{d}{d\tau} \hat{\varphi}(\tau \mid A, a) \bigg|_{\tau = 0} = \frac{1}{i} \varphi(A^* D_a A) \geqq 0$$

for all $a \in \bar{V}_+$ and all $A \in \mathfrak{A}^{(1)}$.

[1] The local structure of $\mathfrak{A}$, however, will not be used here.