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Hölder-Banach Space Analysis of the Bethe-Salpeter Equation

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Abstract. We find a Hölder Banach space in which the Bethe-Salpeter equation is a compact integral equation as it stands. We study the properties of the solution in preparation for an analysis of linear field theory models of 3-body amplitudes. In particular we study the properties of the Regge poles of the solution and prove the existence and uniqueness of on mass shell scattering amplitudes.

Introduction

We wish to consider the properties of linear field theory models for three-body amplitudes taking as input solutions of the two-body Bethe-Salpeter (BS) equation [1]. Previous work [2–12] on the BS equation has shown that the integral operator of that equation can always be related to a compact operator. This is achieved usually by either rotation [3] in the complex energy plane or by the use of spectral representations [8, 10], the end product being the discussion of an L^p (Lebesgue) Banach space and the elimination of the most objectionable if not all singularities of the kernel. This is essential for numerical solution but results in an unnecessarily complicated discussion of the mathematical structure of the equation. In this paper we prove that the BS operator is compact as it stands. We follow the idea of Faddeev [13] and look for a suitable Hölder Banach space for which the assertion is true. We find such a space by first examining the spectral representation introduced by Wick [3] and exploited by Pagnamenta and Taylor [10]. The BS equation written as an equation for the spectral functions (Eq. (1.14)) has many features in common with the non-relativistic Lippmann-Schwinger equation [14] after removal of certain singularities by multiplicative similarity transformations (relations (2.2) and (2.4)). A suitable space in which to look for solutions of Eq. (1.14) is a simple generalisation of that introduced by Faddeev. This Banach space is denoted by $\bar{B}^{(2)}_{\mu\theta}$ with norm specified in relation (2.6). Eq. (1.14) is a compact integral equation in this space. The original BS equation is compact in the space $\tilde{B}^{(2)}_{\mu\theta}$ with norm specified in relation (2.7) generated from $\bar{B}^{(2)}_{\mu\theta}$. Construction of $\tilde{B}^{(2)}_{\mu\theta}$ from $\bar{B}^{(2)}_{\mu\theta}$ has the advantage that functions in $\tilde{B}^{(2)}_{\mu\theta}$ have sufficient analyticity for us to Wick rotate the BS equation for values of total