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Nets of C^*-Algebras and Classification of States*

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Abstract. The concept of locality in quantum physics leads to mathematical structures in which the basic object is an operator algebra with a net of distinguished subalgebras (the "local" subalgebras). Such nets provide a classification of the states of this algebra in equivalence classes determined by local or asymptotic properties. The corresponding equivalence relations are natural generalizations of the (more stringent) standard quasi-equivalence relation (they are also useful for classifying states by their properties with respect to automorphism groups). After discussing general nets from this point of view we investigate in the last section more specialized nets (funnels of von Neumann algebras) with special emphasis on their locally normal states.

Introduction

In the algebraic approach to Quantum Field Theory or Statistical Mechanics one deals with a C^*-algebra $\mathfrak{A}$ with a distinguished collection of subalgebras $\mathfrak{A}_\alpha$. The physical significance of the index α is usually to specify a region in Minkowski space (resp. Euclidean space). Then $\mathfrak{A}_\alpha$ is the algebra generated by the physical operations (or observables) which can be performed in the specified region. The collection $\{\mathfrak{A}_\alpha\}$ provides a "net" for $\mathfrak{A}$ in the sense of Definition 2 below and for many purposes we may assume that it is a "funnel" (see Definition 7).

Parallel to observables and operations we have to consider the physical states. In the mathematical frame they are given by positive linear forms (expectation functionals) over the algebra. The set of these forms is denoted by $\mathfrak{A}^{*+}$. One may take the attitude that each $\omega \in \mathfrak{A}^{*+}$ corresponds to a physical state, but that no actual experimental arrangement can prepare a state precisely. Rather an experiment specifies a weak neighborhood in the space of positive linear forms. This is the point

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