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Renormalizable Models with Simple Symmetry Breaking

I. Symmetry Breaking by a Source Term

K. SYMANZIK
Deutsches Elektronen-Synchrotron DESY, Hamburg

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Abstract. If to a Lagrangian density with invariance under a continuous group of linear transformations of the fields a term linear or bilinear in the fields is added, the symmetry is in general reduced and the currents associated with the original symmetry are only partially conserved. If the theory without the added term is renormalizable, the theory with that term also is, and the needed renormalization conditions are the essential content of the appropriate Ward-Takahashi-Kazes-Rivers identities. The case of symmetry breaking by a term linear in Bose fields (source term) is here analysed completely, in particular with respect to the nonsymmetric limit of vanishing source term, a particular Goldstone mode, and with respect to properties of the ground state energy density as a function of the strength of the source term. Induced and spontaneous breaking of a discrete symmetry are also treated.

Introduction

B. W. Lee [1] has discussed the sigma model [2, 3] from the point of view of renormalized perturbation theory, in order to have available a model that satisfies PCAC[1] and allows to calculate in a formal but consistent way the amplitudes for processes involving nonsoft pions.

We shall show here[2] that for such models the renormalized perturbation expansions can be very simply obtained if the relations, stemming from PCAC, between vertex functions of different numbers of arguments are exploited. These relations yield all the renormalization conditions required in Bogoliubov-Parasiuk-Hepp (BPH)[3] renormalization theory in terms of only that many parameters as the unrenormalized Lagrangian has. This technique also covers the Goldstone mode[4] obtained in the limit of vanishing source but, since it deals with renormalized quantities only,

[1] Ref. [4] discusses the sigma model and related models from the point of view of applications to pion physics.

[2] A short account was given in Ref. [5].

[3] See Ref. [6] and references given there.

[4] Ref. [7] gives a comprehensive presentation of the relevant material.