Quantum Stochastic Processes

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Abstract. In order to describe rigorously certain measurement procedures, where observations of the arrival of quanta at a counter are made throughout an interval of time, it is necessary to introduce the concept of a quantum stochastic process. While fully quantum mechanical in nature, these have a great deal of similarity with classical stochastic processes and can be characterized by and constructed from their infinitesimal generators. The infinitesimal generators are naturally obtained from certain "fields" which we prove must be of the boson or fermion type.

§ 1. Introduction

In the generally accepted mathematical accounts of quantum mechanics the evolution of quantum systems is divided into two basically different types. In the first, the system is not observed and evolves according to a one parameter (time) unitary group of automorphisms of the given Hilbert space. The fact that this is taken to be a unitary group rather than a representation of the positive real line by isometries reflects the belief that the mathematical description of the evolution is essentially unchanged by time inversion. The second kind of evolution occurs during the process of measurement, which is supposed to occur at an instant of time determined by the experimenter. If the state of the system immediately before the measurement is represented by the trace class operator ϱ and the observable is described by a self-adjoint operator

 $A = \sum_{n=1}^{\infty} \lambda_n P_n$ with discrete spectrum, then the state immediately after

the measurement is generally taken to be $\varrho' = \sum_{n=1}^{\infty} P_n \varrho P_n$. The map $\varrho \to \varrho'$ takes pure states to mixed states and time inversion is impossible because $\varrho \to \varrho'$ is not one-one. This is generally explained by the fact that measurement is an essentially irreversible process.

In recent years a class of optical experiments has been performed in which neither of these types of evolution is appropriate but where something very much more complicated seems to be required. The first of these experiments, performed by Hanbury, Brown, and Twiss, and later developments, described in [1, 2], were of the following type. A beam

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