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A Remark Concerning the Charge Operator in Quantum Electrodynamics

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Abstract. The convergence of the integral over the local charge density toward the global charge is investigated within the framework of quantum electrodynamics.

1. Introduction

In relativistic quantum field theories one frequently considers operators which are formal integrals over the entire three dimensional space of the zero component of a conserved quantity. In particular, one writes e.g. for the charge formally

$$Q = \int j_0(x)\, d^3\boldsymbol{x}\,, \qquad \partial^\nu j_\nu(x) = 0\,.$$

Recently one has learned that in case the theory does not contain states of arbitrarily small energy-momentum above the vacuum state this expression is to be understood in the sense

$$(\psi \mid Q\,\varphi) = \lim_{r\to\infty} (\psi \mid Q_r\,\varphi)\,, \tag{1}$$

$$Q_r = \int j_0(x)\, f_r(\boldsymbol{x})\, \alpha(x^0)\, d^3\boldsymbol{x}\,, \tag{2}$$

$$f_r(\boldsymbol{x}) = f_0\left(\frac{\boldsymbol{x}}{r}\right), \qquad r \geqq 1\,, \tag{3}$$

$$f_0(\boldsymbol{x}) = \begin{cases} 1, & |\boldsymbol{x}| \leqq 1\,, \\ 0, & |\boldsymbol{x}| \geqq 2\,, \end{cases}$$

$$\int \alpha(x^0)\, dx^0 = 1 \tag{4}$$

with $f_0(\boldsymbol{x}) \in \mathscr{D}(\boldsymbol{R}^3)$, $\alpha(x^0) \in \mathscr{D}(\boldsymbol{R}^1)$. (The notation is explained at the end of this introduction.) ψ and φ are not arbitrary vectors in the Hilbert space but are generated from the vacuum state by arbitrary local operators [1–4]. This is from the mathematical point of view a rather weak kind of convergence. Strong or weak convergence in the usual sense cannot occur as is shown e.g. in [3] and [4]. On the other hand the result seems to be rather reasonable from the point of view of physics.