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On the Implementability of Automorphism Groups

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Abstract. Let $\mathscr{A}$ be a C^*-algebra and G be a locally compact group acting as strongly continuous automorphisms on $\mathscr{A}$. Let π be a representation of $\mathscr{A}$ then we say π is a covariant representation if there exists a strongly continuous unitary representation of the group acting on $\mathscr{H}_\pi$ which implements the automorphisms. We give necessary and sufficient conditions on a representation π of $\mathscr{A}$ such that a) π is subrepresentation of a covariant representation and b) π is subrepresentation of a covariant representation quasi-equivalent to π.

I. Introduction

Almost every physical problem is connected with the action of some group, sometimes as a symmetry-group of the problem, very often as the group of time development and not seldom as the combination of both. This is also true in those cases where C^*-algebras have been used for the description of physics, as for instance quantum-field-theory and statistical mechanics. In these cases such groups appear as automorphism-groups of the C^*-algebra. Although it is widely believed that all physical information should be purely algebraic, this means, independent of the special representation [1], everyone prefers to use special representations adapted to the special situation. Common to all such representations which have been used so far is the property that the group of automorphisms which are of physical interest are implemented by a continuous unitary representation of the group in question.

At this point we interrupt the discussion in order to introduce some terminology. We denote the C^*-algebra by $\mathscr{A}$ (for the definition see J. Dixmier [2]) and by G a locally compact group. We assume that we have a representation $\alpha : G \rightarrow \operatorname{Aut}\mathscr{A}$ of this group as automorphisms acting on $\mathscr{A}$ ($\operatorname{Aut}\mathscr{A}$ denotes the automorphism-group of $\mathscr{A}$). Speaking about representations π of $\mathscr{A}$ we mean always representations which are not degenerated.

I.1. Definitions. A representation π of $\mathscr{A}$ is called

a) *covariant* if there exists a strongly continuous unitary representation $\varrho : G \rightarrow \mathscr{B}(\mathscr{H}_\pi)$ (all bounded operators on $\mathscr{H}_\pi$) such that $\pi(\alpha_g x) = \varrho(g)\,\pi(x)\,\varrho^{-1}(g)$ for all $x \in \mathscr{A}$;