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# An Analytic Representation of Quantum Field Theory

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**Abstract.** The connection between a space of quadratically integrable functions of real variables $q$ and a Hilbert space of analytic functions of complex variables $z$ established by BARGMANN is used to introduce quantised field operators for which the $\delta$-functions of the commutation relations in $q$-space are replaced by analytic kernel functions in $z$-space, and a reference to distributions can be avoided. BARGMANN's representation is first somewhat modified, so that the derivative terms in the field equations retain their form in the new representation. Local interaction terms in $q$-space obtain a non-local appearance in $z$-space. The transition to a 4-dimensional formulation in $z$-space has to resort to a Euclidean metric. The equations can be derived directly by starting from an action integral in $z$-space, and applying a variational calculus in which variations are restricted to analytic functions. Explicit analytic expressions are given for free field propagators.

## Introduction

BARGMANN established in detail a correspondence between a space of quadratically integrable functions of real variables and a Hilbert space of analytic functions [1]. If one associates complex variables with creation operators of harmonic oscillators, the connection of analytic functions of these variables with wave functions in configuration space has been known for some time, but the scalar product of the analytic function space in BARGMANN's representation and the related kernel functions were new in quantum mechanics, and may still reveal unexplored relationships.

BARGMANN [2], GLAUBER [3], KLAUDER [4] and others have attempted a generalisation of the related concepts to the case of an infinite number of complex variables connected with the description of a quantised boson field in terms of infinitely many oscillator components. This involves the interesting concept of "coherent states", but is not the line to be followed in the present work.

The rather different approach to connect spaces of analytic functions with operators of quantum field theory to be followed here, is based on the simple feature of Bargmann's representation that the $\delta$-function kernel of the unit operator is replaced by a reproducing kernel which is an analytic function of its variables. If one succeeds in replacing the $\delta$-functions in the commutation relations of quantised field operators by