Commun. math. Phys. 11, 339—345 (1969)

Some Remarks on the Ground State of Infinite Systems in Statistical Mechanics

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Received October 25, 1968

Abstract. We investigate the ground states of infinite quantum lattice systems. It is shown in particular that a positive energy operator is associated with these states.

1. Introduction

In a series of recent papers[1] a new approach has been developped for the study of the equilibrium states of infinite systems in statistical mechanics[2]. Classical and quantum lattice systems, and classical continuous systems of particles with hard cores have been considered; their equilibrium states at temperature $T \neq 0$ have been investigated. The present note describes the zero temperature states, i.e. the ground states, of the same systems.

Many of the results which we shall present have proofs similar to and simpler than already published proofs for the case $T \neq 0$. We shall omit these proofs, and present therefore a list of theorems mostly without proofs. It will be remarked however that our results about the ground state are not special cases of results for $T \neq 0$, and that some of them have in fact no obvious counterpart at $T \neq 0$.

2. Infinite Volume Limit for the Ground State

It will be convenient to work with quantum lattice systems, but the results obtained in this section extend to classical lattice gases and classical continuous systems of particles with hard cores (see Footnote 1).

We let $\mathscr{H}$ be a complex Hilbert space with finite dimension and $\mathscr{H}_x$ a copy of $\mathscr{H}$ at each point x of the "lattice" $\boldsymbol{Z}^\nu$. For finite $\Lambda \subset \boldsymbol{Z}^\nu$, let

$$\mathscr{H}(\Lambda) = \bigotimes_{x \in \Lambda} \mathscr{H}_x$$

[1] See ROBINSON and RUELLE [8], GALLAVOTTI and MIRACLE [2], RUELLE [10], LANFORD and ROBINSON [5], ROBINSON [7], GALLAVOTTI and MIRACLE [3], LANFORD and ROBINSON [6]. A general treatment is also given in a forthcoming book [11].

[2] Some of the ideas involved appear already in RUELLE [9] and FISHER [1].