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# A Generalization of Dyson's Formula

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**Abstract.** We give an integral representation for tempered distributions which have more general support properties in $x$ space, than those usually assumed in the derivation of the Dyson formula.

The existence of such an integral representation is shown to be equivalent to that of a suitable extension of an analytic function: namely, given an analytic function on a section $\Omega$ of a domain of holomorphy $\Omega'$ extend it to $\Omega$ imposing on it some growth conditions.

$L^2$ space methods of L. HÖRMANDER are used to solve this problem of extension. In order to apply these Hilbert space techniques, it was necessary to prove two important theorems on the growth of analytic function.

From the physical point of view the formula we obtained is an integral representation for the commutator of two quasi local fields.

## Introduction

Let us consider the Fourier transform of the commutator of two operators in the Haag Araki theory:

$$f(p) = \int e^{i(p,x)} \left\langle \Omega \left| \left[ A\left(\frac{x}{2}\right), B\left(-\frac{x}{2}\right)\right] \right| \Omega \right\rangle d^4x . \tag{0}$$

In the above formula, $A$ and $B$ are local observables contained respectively in the von Neuman algebras $R(\mathscr{B}_A)$ and $R(\mathscr{B}_B)$.

$$\begin{aligned} A(x) &= U(x)\, A\, U(-x) \\ B(x) &= U(x)\, B\, U(-x) \\ U(x) &= e^{i(P,x)} , \end{aligned}$$

$P$ is the energy momentum operator [4], the scalar product $(p, x)$ means $p_0x_0 - p_1x_1 - p_2x_2 - p_3x_3$, and $\Omega$ is the vacuum state.

The tempered distribution $f$ has the following properties:

a) its support is contained outside a region $\mathscr{R}$ which is roughly bounded by two spacelike surface (for a precise description look at Refs. [0, 1, 2] and especially pages 323–325 of Ref. [5]).

b) According to the causality condition, its Fourier transform vanishes on the set[1]:

$$\left\{ x \in \mathbb{R}_4 \left| \frac{x}{2} + \mathscr{B}_A \sim -\frac{x}{2} + \mathscr{B}_B \right. \right\} .$$

[1] Let $S$ and $T$ be sets in $\mathbb{R}_n$, then $S + T = \{x \in \mathbb{R}_n | x = s + t; s \in S, t \in T\}$; for any $x, y \in \mathbb{R}_4$

$x \sim y$ means $(x - y)^2 = (x_0 - y_0)^2 - (x_1 - y_1)^2 - (x_2 - y_2)^2 - (x_3 - y_3)^2 < 0$ .