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Analyticity Properties of the Anisotropic Heisenberg Model

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Abstract. An upper bound $\bar{T}_c$ for the critical temperature of a class of spin systems which includes the Heisenberg ferromagnet is derived. The analyticity of the free energy as a function of the temperature, the external magnetic field, and the interaction potentials, is demonstrated in a domain which includes all temperatures $T > T_c$. For the isotropic Heisenberg ferromagnet in ν dimensions we find the poor estimate $2\nu J/k\bar{T}_c = 0.0001$. Some analyticity and cluster properties of the reduced density matrices are also derived.

1. Introduction

It was first shown by RUELLE [1, 2] that, in the low density region, one could derive analyticity and cluster properties for the correlation functions of a classical statistical mechanical system from the Kirkwood-Salzburg integral equations. His method consisted in interpreting the latter equations as integral equations on a suitably chosen Banach space. The method was developed by GINIBRE [3] who obtained similar results for continuous quantum systems and, more recently [4], for certain quantum spin systems or lattice gases. The idea behind GINIBRE'S innovation is to reduce the quantum mechanical problem to a problem formally identical to the classical one by the use of Wiener integral techniques. These latter techniques allow one to represent the quantum mechanical reduced density matrices in terms of classical correlation functions over a space of quantum mechanical configurations which physically consist of "clouds" of classical configurations.

In the case of classical lattice systems RUELLE'S method was generalized to incorporate many-body forces [5] and improved to give a much larger region of analyticity in terms of the thermodynamical variables [6, 7].

The latter improvement originates from two sources. Firstly one remarks that by taking into account the presence of a hard core condition and the fact that the configurations form a discrete set one may