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Reduced Density Matrices of the Anisotropic Heisenberg Model*

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Abstract. The reduced density matrices of the anisotropic Heisenberg model are studied by means of a functional integral representation based on a generalized Poisson process. Integral equations, which are analogous to the classical Kirkwood-Salzburg equations, are then used to prove the existence of the infinite volume limit of the reduced density matrices, analyticity properties with respect to the fugacity (or magnetic field) and the potentials, and a cluster property, in the low fugacity (high magnetic field) region.

Introduction

The correlation functions of classical gases and their quantum analogues have been studied recently by a method due to Ruelle [1]. This method rests on the well known Kirkwood-Salzburg equations [2] and various generalizations thereof, and has been used to prove the existence of the infinite volume limit, some analyticity properties, and a cluster property of the correlation functions for sufficiently small fugacities. The method has been applied to classical continuous systems [1], to classical lattice systems [3], and to continuous quantum systems [4]. In the latter case, in order to obtain and exploit appropriate generalizations of the Kirkwood-Salzburg equations, one uses an integral representation of the quantum analogues of the correlation functions, namely the reduced density matrices (R.D.M.), based on the Wiener integral.

In the present paper, we want to point out that the same method also applies to a variety of quantum lattice systems. (For general properties of such systems, see ref. [5, 6, 7].) In fact, one can use a functional integral representation of the RDM based on a discrete analogue of the Wiener process, which turns out to be a generalized Poisson process. Various models can be studied, some of which are listed in the last section of this paper. Meanwhile, we concentrate on one of them, namely the anisotropic Heisenberg model, which is both of current interest and typical of the scope of the method.

In Section 1, we define the model, describe the associated stochastic process, and give some estimates which are needed in subsequent proofs. In Section 2, we give the definition and integral representation of the

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