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On Some Representations of the Anticommutations Relations

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Abstract. We study representations of the canonical anticommutation relations having the form:

$$A(f) = a(Hf) + b^*(Kf)$$
$$A^*(f) = a^*(Hf) + b(Kf)$$

where $a(f)$, $b^*(f)$ and their adjoints are two basic anticommuting fields in a Fock Space.

A complete determination of the type in terms of $|K| = (K^*K)^{1/2}$ and a sufficient condition for quasi-equivalence are given.

I. Introduction

Let $\mathfrak{E}$ be a complex Hilbert space of test functions, denoted by $f, g, h, \ldots$ To each element f of $\mathfrak{E}$ correspond two bounded operators on a Hilbert space $\mathfrak{F}$, $a(f)$ and $b^*(f)$, depending linearly and continuously on f in the uniform topology of operators. We denote briefly their adjoints by $a^*(f)$ and $b(f)$; therefore, these are semi-linear in f. We impose the relations:

$$\begin{gathered}[a(f), a(g)]_+ = [b^*(f), b^*(g)]_+ = [a(f), b^*(g)]_+ = [a(f), b(g)]_+ = 0 \\ [a(f), a^*(g)]_+ = [b(g), b^*(f)]_+ = (f, g) \\ f, g \in \mathfrak{E}, \quad [A, B]_+ = AB + BA ,\end{gathered} \tag{1}$$

and we take for $\mathfrak{F}$ the customary Fock-space associated with these two anticommutating fields. Id est, we have in $\mathfrak{F}$ a vector Ω_0 such that:

$$a(f)\Omega_0 = b(g)\Omega_0 = 0 , \quad f, g \in \mathfrak{E} \tag{2}$$

and all the linear combinations of vectors having the form:

$$a^*(f_1) \ldots a^*(f_m)\, b^*(g_1) \ldots b^*(g_n)\Omega_0$$

are a dense set in $\mathfrak{F}$.

Now, if H and K are operators in $\mathfrak{L}(\mathfrak{E})$ which satisfy:

$$H^*H + K^*K = I \tag{3}$$

we set:

$$A(f) = a(Hf) + b^*(Kf) . \tag{4}$$