Commun. math. Phys. 9, 169—175 (1968)

BCS-Spin-Model, its Thermodynamic Representations and Automorphisms

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Received April 1, 1968

Abstract. In this paper two equivalent explicit realizations of the thermodynamic representations of the BCS-spin-model are constructed and since they are of well known form their type is easily determined. Time evolution and vacuum properties of these representations are studied and it is shown that in general the time development of the system is representation-dependent.

Our starting point is the Hamiltonian

$$H_N = \varepsilon \sum_{p=1}^{N} (1 - \sigma_p^z) - \frac{2T_0}{N} \sum_{pp'=1}^{N} \sigma_p^- \sigma_{p'}^+ \tag{1}$$

where the first term describes the energy of Cooper-pairs and the second their interaction. ($\boldsymbol{\sigma}_p = (\sigma_p^x, \sigma_p^y, \sigma_p^z)$ are the usual Pauli matrices $\sigma_p^{\pm} = (\sigma_p^x \pm i\sigma_p^y)/2$). For $N \to \infty$ the thermal expectation values of products of $\boldsymbol{\sigma}_p$'s simplify and just these limits are used for physical calculations. The price one has to pay for this simple structure of the thermodynamic functional are the mathematical difficulties connected with infinite particle numbers.

Let Σ_1 be the smallest *-algebra generated by the $\boldsymbol{\sigma}_p$'s. We suppose the elements of Σ_1 to be realized in a natural way as bounded linear operators in the Hilbert-space $\mathfrak{H} = \bigotimes_{p=1}^{\infty} \mathfrak{H}_p$ (all $\mathfrak{H}_p$ two-dimensional) [1]. In this representation one takes the uniform closure (in the sense of the usual operator norm $\|A\| = \sup_{\|x\|=1} \|Ax\|\ x \in \mathfrak{H}$) of Σ_1, we call the resulting C^*-algebra Σ.

Now we state a result: Let A be an element of Σ_1 and

$$\langle A \rangle_\beta = \lim_{N\to\infty} \langle A \rangle_{N,\beta} = \lim_{N\to\infty} \frac{\operatorname{tr}(e^{-\beta H_N} A)}{\operatorname{tr}(e^{-\beta H_N})} \tag{2}$$

where the trace for fixed N goes over the 2^N-dimensional space on which the (only) irreducible representation of the system $\{\boldsymbol{\sigma}_1, \ldots \boldsymbol{\sigma}_N\}$ acts. For $\beta \leqq \beta_0 = \varepsilon^{-1} \operatorname{arcth} \varepsilon/T_0$ one gets

$$\langle \sigma_{p_1}^{(i_1)} \ldots \sigma_{p_m}^{(i_m)} \rangle_\beta = (\operatorname{th} \beta\varepsilon)^m\, n^{(i_1)} \ldots n^{(i_m)} \quad \text{with} \quad \boldsymbol{n} = (0, 0, 1) \tag{3}$$